Unit-roots and broken trends

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Traditional view of the economic cycle:

\[ y_t = a + bt + \varepsilon_t \]

Trend stationary (TS): a stationary fluctuation \( \varepsilon_t \) around the time trend \( a + bt \). \( y_t \) is obviously non-stationary. Stationarity is easily achieved by ‘removing the trend’

Competing model:

\[ y_t = a + y_{t-1} + \varepsilon_t \]

Random walk with drift (RWD): It is a difference stationary (DS) process. This model represents the ‘unit-root hypothesis’.
Nesting model:

\[ y_t = a + bt + \phi y_{t-1} + \varepsilon_t \]

The null hypothesis that the process is characterized by the random walk can be evaluated by \( \phi = 1 \) in the OLS estimation using the Dickey-Fuller procedure.

- Nelson and Plosser (1982): most US macroeconomic time series seems to be better characterized by a unit-root process (accept null).

- Perron (1989): a more appropriate model is:

\[ y_t = a + d_t + bt + \varepsilon_t \]

where:

\[ d_t = \begin{cases} 
  a) 1 & \text{if } t < k, \ 0 \text{ otherwise} \\
  b)t - k & \text{if } t \geq k, \ 0 \text{ otherwise}
\end{cases} \]

that is, a trend-stationary model in which the variable \( d_t \) allows for a deterministic break in the trend at moment \( k \).

- Case (a): change in intercept of the trend (shift in mean)
- Case (b): change in slope of the trend without any sudden change in the moment of the break (shift in trend).
• All innovations have temporary effect except for the one that changes the trend at time $k$, and it turns out to be the only shock in the process that has a permanent effect

• Standard unit root tests confuse this with a unit root and hence is biased towards accepting the null!!.

• *Perron’s transformed uroot test*: account for break and test. The null of a unit-root can be rejected in most cases in the Nelson-Plosser data when a break is allowed

Doubts on Perron’s conclusion

• Break point ($k$) is determined exogenously in Perron's analysis.

• If this is influenced by observation of the data, it introduces a pre-test bias that favors rejecting the no-break null hypothesis.

• Intuition: choice of a breakpoint based on prior observation of the data is inconsistent with a testing procedure based on a distribution that is claimed to be data independent.

*Solution*: a data-dependent algorithm to find the break-point endogenously and then to evaluate the unit-root hypothesis using distributional results that take into account this data-dependence to avoid the pre-test bias.
Test statistics (Banerjee, et.al.(1992))

ADF regression:

\[ y_t = a + bt + \phi y_{t-1} + \beta(L)\Delta y_{t-1} + \varepsilon_t \]

\(\beta(L)\) is a lag polynomial of order \(p\).

- 1. **Recursive tests:** recursive DF \(t\) statistic with subsamples \(t = 1, \ldots, k\) where \(k = k_0, k_0+1, \ldots, T\). Evaluate the maximum and minimum DF test.

- 2. **Rolling tests:** DF based on subsample of fixed size \(T_s\), rolling through the sample. Evaluate maximum and minimum DF.

- 3. **Sequential tests:** estimate

\[ y_t = a + bt + cd_t + \phi y_{t-1} + \varepsilon_t \]

allowing for a possible single shift or break at every point in the sample.

Statistics considered are:
- Maximum \(F\)-statistic evaluating \(c = 0\).
- DF statistic evaluated at the period in which the \(F\)-statistic evaluating \(c = 0\) is maximum.
- Minimum DF statistic evaluating \(u = 1\)

Two sets of sequential statistics are evaluated, one allowing for a change in mean (case (a)) and another one allowing for a change in the slope of the trend (case (b)).
Empirical results for Argentina

TABLE 1: Summary of Statistics. Annual Data (1900-1993)

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Period</th>
<th>Crit. Val (10%)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Full Sample Statistics</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-DF</td>
<td>-2.8339</td>
<td>-3.15</td>
</tr>
<tr>
<td>-Phillips-Perron</td>
<td>-2.9132</td>
<td>-3.15</td>
</tr>
<tr>
<td><strong>Recursive</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Min DF</td>
<td>-3.6293</td>
<td>1980</td>
</tr>
<tr>
<td>-Max DF</td>
<td>-1.9619</td>
<td>1934</td>
</tr>
<tr>
<td><strong>Rolling</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Min DF</td>
<td>-4.3392</td>
<td>1964</td>
</tr>
<tr>
<td>-Max DF</td>
<td>-0.3382</td>
<td>1989</td>
</tr>
<tr>
<td><strong>Quandt test</strong></td>
<td>14.3100</td>
<td>1917</td>
</tr>
<tr>
<td></td>
<td></td>
<td>23.86</td>
</tr>
<tr>
<td><strong>Trend-Shift</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Max F</td>
<td>11.0637</td>
<td>1981</td>
</tr>
<tr>
<td>-DF at F max</td>
<td>-4.3984</td>
<td>1981</td>
</tr>
<tr>
<td>-Min DF</td>
<td>-4.3984</td>
<td>1981</td>
</tr>
<tr>
<td><strong>Mean Shift</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>-Max F</td>
<td>6.8685</td>
<td>1975</td>
</tr>
<tr>
<td>-DF at F max</td>
<td>-3.9269</td>
<td>1975</td>
</tr>
<tr>
<td>-Min DF</td>
<td>-3.9269</td>
<td>1975</td>
</tr>
</tbody>
</table>

- **DF and Phillips Perron:** do not reject the null.
- **Recursive:** minimum and maximum DF statistics are -3.6293 and -1.9619 at 1980 and 1934 respectively. Do not reject null.
- **Rolling:** DF statistics are -4.3392 and -0.3382 at periods 1964 and 1989 respectively. Do not reject null.
- **Sequential statistics:** Change in the intercept of the trend (case (a)): maximum F is observed in 1981, with value 11.0637, lower than the critical value reported by Banerjee, et. al (1992). Hence, we do not reject the null of no change in the intercept of the trend. Sequential DF tests computed under case (a). The minimum DF test gives a value of -4.3984, greater than the critical value in Banerjee, et al. (-4.52) suggesting not rejecting the null hypothesis of a unit-root. This minimum DF test is observed at the same period in which the F statistic achieves its maximum value. Similar results hold for case (b).
Perron’s modified DF test (break determined exogenously) If we treat the break periods 1975 and 1981 as determined exogenously for cases (a) and (b), then our DF statistics are both lower than Perron’s critical values at 10% suggesting that the unit-root hypothesis should be rejected.

Results are consistent recent findings: after we eliminate the pre-test bias using distributional results consistent with a data-dependent time break the unit-root hypothesis cannot be rejected.

Conclusion: The null hypothesis of the presence of a unit-root in the stochastic process governing argentine real GDP cannot be rejected using a conservative size of 10%. In all cases we considered the possibility of an endogenously determined break in the deterministic trend as a possible characterization.